

PROOF THAT IN FIELD ELECTRON AND FIELD ION MICROSCOPES THE ELECTRON/ION PATH IS INDEPENDENT OF THE CHARGE/MASS QUOTIENT

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SUMMARY

In a given field ion microscope (FIM), different operating gases – with different masses and different values of their charge-to-mass quotients – follow nearly exactly the same trajectories, from a given atomic site on the surface of a given emitting needle to the ion detector. This is an EXPERIMENTAL FACT that has been known to FIM operators for over 50 years, but seems incompatible with usual theoretical thinking about charged-particle motion, and seems not to have received a clear theoretical explanation. This Poster reports an analytical theoretical-physics explanation, based on Newton's and Coulomb's Laws. It also follows that, if emitter polarity is reversed, then emitted electrons are expected to follow nearly exactly the same trajectories as the gas ions – which is an effect that has been observed in finite-element trajectory simulations and is consistent with old observations with field ion and field electron (emission) microscopes.

1. INTRODUCTION

This Poster relates to the basic **classical charged-particle (CP) optics** of the **field ion microscope (FIM)** and the **field electron microscope (FEM)**. Figure 1 is a (very) schematic diagram of a FIM/FEM system, and Fig. 2 is an enlarged diagram relating to CP trajectories near a needle-like "field emitter".

Consider a point-like charged particle that is emitted, with zero velocity, from an off-axis position on the emitter surface, such as "P". Initially, this CP moves normally to the emitter surface. However, the emitter shank acts as a weak converging lens and the CP trajectory bends inwards [1,2].

The question of interest for this paper is the following. If classical charged particles with different values of their **charge-to-mass quotient (q/m)** start from the same position on the smooth classical surface of a needle-shaped emitter, with zero initial velocity, do they follow the exactly the same trajectory and end up at exactly the same position on the detector?

Field ion microscopes can be operated with different operating gases (H_2 , He, Ne, etc.) that have significantly different values of their charge-to-mass quotients. Above any particular surface atom, operating-gas entities of different chemical kinds start from nearly exactly the same position in space, with initial kinetic energies that are similar for all gases and very small in comparison with the energy acquired by the time they hit the detector.

One of us (RGF) did an experimental PhD [3] in field ion microscopy long ago, but remains familiar with the details of the technique. It is an **experimental fact**, known to older FIM users but poorly documented in the published literature, that FIM images at "best image voltage" have much the same *general* size and appearance for different operating gases.

It has also recently been established [4], by means of finite-element numerical simulations, that H_2^+ ions and electrons that start with zero velocity from the same point on a classical needle-shaped emitter, do (within the error limits of the simulation) follow exactly the same trajectory and end up at exactly the same point on the detector. This result was also found earlier [5] by one of us (TAdA).

Such evidence as currently exists concerning experimental FEM and FIM images of basically the same endform tends to be consistent with this finding.

This behaviour is at variance with usual thinking about the behaviour of classical charged particles in electromagnetic fields and (to the best of our knowledge) has never received a good first-principles explanation. This Poster seeks to provide at least a first step towards such an explanation.

2. MATHEMATICAL THEORY:

2A: PARTICLE TRAJECTORIES IN A PURELY ELECTROSTATIC FIELD

The motion of a charged particle (CP) in a **purely electrostatic (ES) field $E(r)$** is governed by the Laws of **Newton** and **Coulomb**, via the equation:

$$m \frac{d^2 r}{dt^2} = qE(r), \quad (1)$$

where r is the **CP position** as a function of **time t** .

Now introduce a **rescaled time variable τ** defined by

$$\tau \equiv (q/m)^{1/2} t. \quad (2)$$

This allows us to rewrite the equation of motion by rescaling the time derivatives via the relations

$$\frac{dr}{dt} = \left(\frac{dr}{d\tau}\right) \cdot \left(\frac{d\tau}{dt}\right) = (q/m)^{1/2} \left(\frac{dr}{d\tau}\right), \quad (3)$$

$$\frac{d^2 r}{dt^2} = \left(\frac{d^2 r}{d\tau^2}\right) \cdot \left(\frac{d\tau}{dt}\right) = (q/m) \cdot \left(\frac{d^2 r}{d\tau^2}\right). \quad (4)$$

The equation of motion thus becomes

$$\frac{d^2 r(\tau)}{d\tau^2} = E(r). \quad (5)$$

Clearly, this equation does not involve **q , m or q/m** .

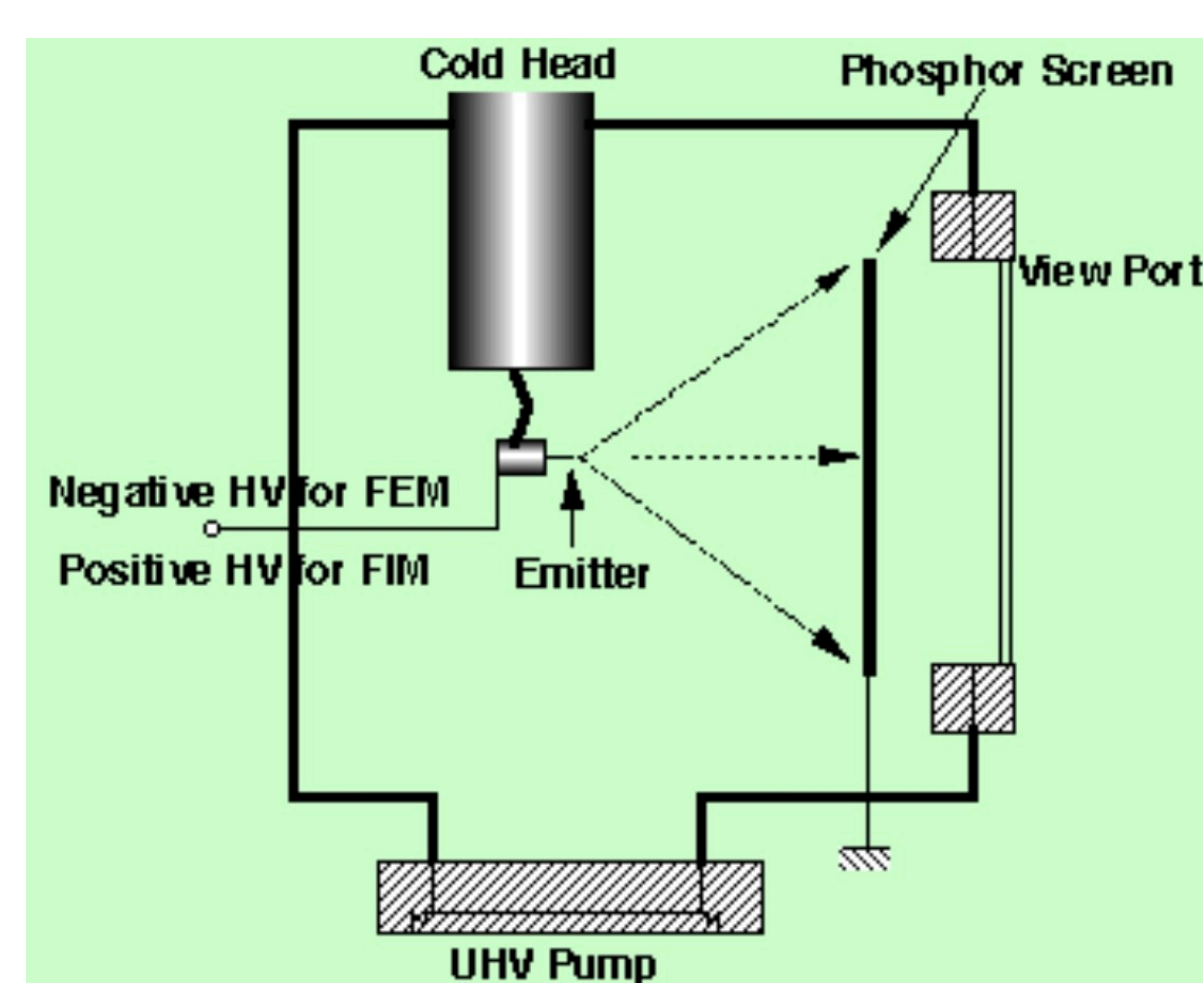


Fig. 1: Schematic diagram to illustrate the operation of the field ion and field electron microscopes. (For FIM an operating-gas supply is also needed.)

[Courtesy Wikipedia]

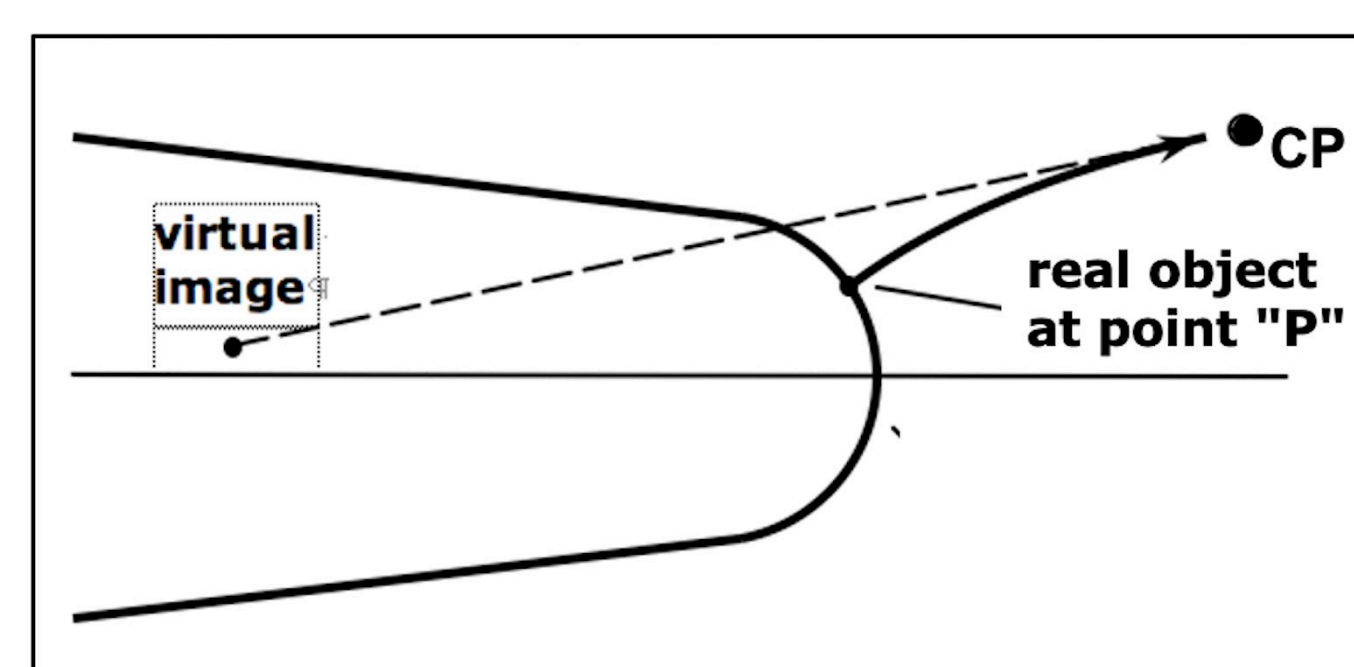


Fig. 2: To illustrate the trajectory of a charged particle (CP) emitted normally from point P on the surface of a field emitter, with zero initial velocity.

MATHEMATICAL THEORY (cont.)

If CPs with different **(q/m)** -values start from the same position r_0 in space, **with zero velocity at that position**, then the solutions of eq. (5) will be the same in all cases. In these conditions, CPs with different **(q/m)** -values will follow the same trajectories in space.

However the real-time taken to complete the trajectory *will* depend on the charge-to-mass quotient **(q/m)** .

2B: TWO-ELECTRODE GEOMETRY

A further result exists in two-electrode geometry if all surfaces are treated as having the same work function, and thus the ES potential difference between the electrodes equals the applied **measured-voltage V_m** between them.

A change in **V_m** by a factor λ changes the whole ES-field distribution **$E(r)$** by this factor λ . However, increasing the mass **m** in eq. (1) by the same factor λ reproduces the original equation. Particle trajectories in an ES field are independent of particle mass. It follows that, in two-electrode geometry, trajectories are also independent of the voltage between the electrodes.

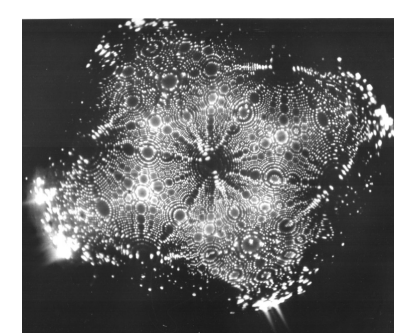
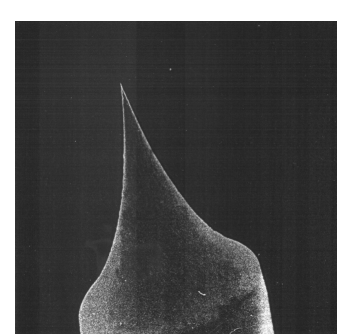
3. CONCLUSIONS

Starting from Newton's and Coulomb's Laws, it has been shown (quite generally) that, in a purely electrostatic field, the trajectory of a classical charged particle is independent of its charge-to-mass quotient **(q/m)** , if it starts with zero velocity from a defined point in space. Further, in an ideal two-electrode system, the trajectory is independent of the measured-voltage **V_m** between the electrodes when the CP starts from the relevant electrode.

We think this result is related to the 1927 Busch work on electron motion in electromagnetic fields that led to the development of electron microscopy. We think this electrostatic result is **not** generally well known, but suspect that: (a) it was in fact proved long ago (possibly not in English): and/or (b) it is well known to some specialised scientific/technological communities. Help in locating an earlier general proof (if any such proof exists) would be welcome.

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